UBC-GIF: Capabilities for EM Modelling and Inversion of LSBB data

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LSBB: Karst Aquifer Characterization

- Principle Questions:
  - What is the underground matrix
  - Hydrogeologic properties
  - Storage capacity
  - Production capacity
  - Potential for pollution
  - Sustainability

- What is the role of geophysics?
Framework for Applied Geophysics

- What is the question to be answered?
- What are the diagnostic physical properties?

  - Density
  - Magnetic susceptibility
  - Electrical conductivity
  - Electrical permittivity
  - Elastic parameters

- Choose survey type
- Collect data
- **Invert data** to get physical property model
- Interpret model and synthesize with other data
Geophysical Experiment

- Physical property: Electrical conductivity
- Survey: DC Resistivity
- Collect data

Data images cannot be directly interpreted in terms of geology
Data must be inverted
What is Inversion?

Inversion processing

Model Inversion estimates Earth models based upon data and prior knowledge.

Measurements over the Earth are data.
The inverse problem

• Geophysical data are: \( F[m] + \epsilon = d \)
  - \( m \): model --- unknown
  - \( F \): forward mapping operator
  - \( \epsilon \): errors
  - \( d \): observations (data)

• Given:
  - data, errors, a forward modelling method

• Find:
  - the model that generated measurements.

• Major Difficulty: Nonuniqueness
Inversion as optimization: 3 parts

**Misfit:**
\[ \phi_d = \sum_{i=1}^{N} \left( \frac{F_i[m] - d_i^{obs}}{\epsilon_i} \right)^2 \]

* \( \epsilon_i \): standard deviation

**A priori** information: reference model, structural detail...

**Model objective function:**
\[ \phi_m(m) = \alpha_s \int_S (m - m_0)^2 \, dv + \alpha_x \int_S \left( \frac{\partial (m - m_0)}{\partial x} \right)^2 \, dv + ... \]

* \( \alpha_s, \alpha_x \) … constants
* \( m_0 \): reference model

**Inversion as optimization:**
\[ \phi = \phi_d + \beta \phi_m. \]

0 < \( \beta < \infty \) is a constant
Choose \( \beta \) such that \( \phi_d < \text{Tolerance} \)
Numerical solution

• Discretize: Divide the earth into M cells of constant physical property (M >> N).

• Minimize

\[ \phi = \phi_d + \beta \phi_m \]

\[ = \left\| W_d (F[m] - d^{obs}) \right\|^2 + \beta \left\| W_m (m - m_0) \right\|^2 \]

• Use the Gauss-Newton method for solution.

• Solving for $\beta$:
  - Discrepancy principle.
  - GCV.
  - L-Curve.
Numerical solution (Gauss-Newton method)

• Minimize \( \phi = \phi_d + \beta \phi_m \)

\[
\begin{align*}
= & \left\| F[m] - d^{obs} \right\|^2 + \beta \left\| W(m - m_0) \right\|^2 \\
\end{align*}
\]

• set

\( g(m) \equiv \frac{\partial \phi}{\partial m} = J(m)^T (F[m] - d^{obs}) + \beta W^T W (m - m_0) = 0 \)

\( J \) is the sensitivity matrix: \( J_{ij} = \frac{\partial d_i}{\partial m_j} \)
Numerical solution (Gauss-Newton method)

- Minimize \( \phi = \phi_d + \beta \phi_m \)
  \[
  = \left\| F[m] - d^{obs} \right\|^2 + \beta \left\| W (m - m_0) \right\|^2
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  \( J \) is the sensitivity matrix: \( J_{ij} = \frac{\partial d_i}{\partial m_j} \)

- Expand forward operator (dropping higher order terms):
  
  \( F[m + \delta m] \approx F[m] + J(m) \delta m \)
Numerical solution (Gauss-Newton method)

• Minimize \( \phi = \phi_d + \beta \phi_m \)
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  \( J \) is the sensitivity matrix:
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  \]

• Expand forward operator (dropping higher order terms):
  \[
  F[m + \delta m] \approx F[m] + J(m) \delta m
  \]

• Solve:
  \[
  \left( J(m_k)^T J(m_k) + \beta W^T W \right) \delta m = g(m_k)
  \]
  • where \( m_k \) is the model at the \( k^{th} \) iteration.
  • This is an \( M \times M \) system of equations.

\((M = \# \text{ model parameters, or cells})\)
Inversion Capabilities

- Gravity (3D)
- Magnetics (3D)
- DC resistivity and IP (2D and 3D)
- Frequency domain EM (1D and 3D)
- Time domain EM (1D and 3D)

Software for inversion is distributed world wide through UBC and third party vendors
Field Example: San Nicolas Deposit

Location

Geologic cross section

Physical properties

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Elevation (m)

-1100

Easting (m)

-2000

1600

2000
Gravity data collected at the San Nicholas deposit.
Gravity Inversion Results

Cross-section of Density Contrast Model with Geology

Density contrast

Easting (m)

Depth (m)

-0.22
0.07
0.35
g/cc

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Magnetics

Magnetic data collected at the San Nicholas deposit.
Magnetic Inversion Results

Cross-section of Magnetic Susceptibility Model with Geology
Field Example: San Nicolas Deposit

Geologic cross section

- Mafic Volcanics
- Tertiary Breccia
- Quartz Rhyolite
- Massive Sulphide
- "Keel"

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- Density
- Magnetic Susceptibility
- Electrical Conductivity
- Chargeability
Electrical Conductivity: Different Surveys

Sources:
- Galvanic (grounded electrodes)
- Inductive (current loops)

Waveforms
- Sinusoidal (Frequency domain)
- Time waveforms (Time domain)
3D EM: Frequency Domain

Source: Loop or grounded electrode

Waveform

I

Time

Data (E, H)

Borehole Data

(E, H)

\[ \nabla \times \mathbf{E} - i\omega \mu \mathbf{H} = 0 \]

\[ \nabla \times \mathbf{H} - (\sigma - i\omega \varepsilon)\mathbf{E} = \mathbf{J}^e \]
CSEM Survey

- 15 Frequencies between 0.5Hz, 8192 Hz
- 3 lines, 1.6km long, 200m apart
- 25 meter station spacing
- single transmitter
- data are scalar impedances (Ex/Hy)
- data collected with the goal of MT interpretation

![Diagram showing transmitter and receivers with distances labeled.]
3D CSEM Inversion

Frequencies
0.5, 8, 64, 256 Hz

Iso-surface cutoff 10 Ohm-m
**3D TEM Setup**

**Source**
(Loop or grounded electrode)

**Surface Data**
$(E, H, dB/dt)$

**Borehole Data**
$(E, H, dB/dt)$

**Waveform**
(half sine, step…)

\[
\nabla \times E + \mu H_t = 0 \\
\nabla \times H - \sigma E - \epsilon E_t = s_r(t)
\]

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Introduction to UTEM data at San Nicolas

- 3 large loop transmitters
  - 2 km by 1.5 km
- dB/dt receivers
  - mainly z component

- transmitter waveform
  - 30 Hz sawtooth wave
  - dI/dt constant over half cycle
San Nicolas UTEM data

UTEM channel 4 (1.513ms)

dBz/dt

nT/s

Loop 9

nortning

1075

1075

-1300 -3000 -220

UTEM channel 4 (1.513ms)
Fitting the data

Observed 15 \( \Omega \text{m} \) iso-surface

View from SW

One decay curve: Observed and predicted

Observed

Predicted
San Nicolas inversion results:

Recovered cross section at 450 S

Resistivity from drilling at 450 S
Density, Magnetic susceptibility, Conductivity

Density contrast

Magnetic susceptibility

3D CSEM

3D UTEM
Field Example: San Nicolas Deposit

Geologic cross section

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- Magnetic Susceptibility
- Electrical Conductivity
- Chargeability
**Induced Polarization**

- Source Current (Amps)
- Measured Voltage (Volts)

Non-zero area occurs because charges took time to equilibrate.

*IP datum: Vs/Vn*

Collect IP data along with DC resistivity data.
DC/IP data at San Nicolas

- Pole-dipole
- Real Section

3D Inversion

Chargeability

Pseudo-section
Summary of physical property inversion at San Nicolás

**Sulfide**: dense, chargeable, susceptible, conductive.
Back to Karst Aquifers: LSBB

- Soil
- Epikarst
- Unsaturated
- Saturate

- What scale?
- Which physical properties?
Back to Karst Aquifers: LSBB

• Tunnel Scale (meters to km):
  – Conductivity (indicative of water)
    (DC resistivity, FEM or TEM)
  – IP might be useful for clay layers if they are chargeable
  – Time-lapse DC (EM) resistivity can provide information about hydraulic conductivity
  – Electrical permittivity (GPR)
Geophysics for large scale aquifer

- Conductivity
  - Airborne EM for 200 meters (Epikarst delineation)
  - ZTEM or MT for deeper structure (large voids/conduits, depth of saturated zone)
Geophysics for large scale aquifer

- Density: (Change in water volume)
- Magnetics ??
- Self-potential (from fluid motion)
- Chargeability ??
- Self-potential, MRI, Seismic

Good news from geophysics side: We can invert most types of survey data to recover 3D distribution of physical properties.
Thank you!