**USE OF A MATHEMATICAL MODEL TO STUDY THE DYNAMICS OF *Ctenocephalides felis* POPULATIONS IN THE HOME ENVIRONMENT AND THE IMPACT OF VARIOUS CONTROL MEASURES**

BEUGNET F.*, PORPHYRE T.**, SABATIER P.** & CHALVET-MONFRAY K.**

**Summary:**

The biology of fleas has been studied by a number of authors, as has the impact of various types of control measures. However, there are no mathematical models simulating the dynamics of a population of *Ctenocephalides felis felis* fleas on their host (the cat) and in their close environment (apartment). The model presented in this paper allows for integration of the numerous biological and behavioural parameters of the parasites and their hosts and for the variation of these same parameters. The various types of control measures can be programmed so that their impact over time can be studied. The model confirms the key role played by adult fleas, or emerged fleas contained in the cocoon. Only regular applications of persistent insecticides to the host animal will enable control of the parasite population. A combination of these insecticides with an IGR (Insect Growth Regulator) will accelerate decontamination of the home environment and see the disappearance of the parasites altogether if they are not reintroduced. The association of additional measures such as vacuum cleaning will accelerate the process of decontamination but will have no impact if carried out in isolation. One-off treatment with insecticide will not enable a reduction in the parasite population, even if carried out frequently. Use of insecticides on the home environment premises alone does not appear to be an adequate means of control. The present model can be used to test various integrated control measures which take into account different factors such as the number of host animals, the frequency of movement outdoors, the impact of the seasons.

**KEY WORDS:** *Ctenocephalides felis*, mathematical model, population dynamics, insecticide, IGR (Insect Growth Regulator).

**Résumé:** Modélisation de la dynamique de populations de *Ctenocephalides felis* dans l'environnement domestique et étude de l'impact des différents moyens de contrôle

La biologie des puces a été étudiée par de nombreux auteurs, comme les différentes modalités de traitement. Il n’existe cependant pas de modèle mathématique simulant la dynamique d’une population de puces *Ctenocephalides felis felis* sur son hôte (chat) et dans l’environnement proche (de type appartement). Le modèle ici présenté permet d’intégrer et de faire varier les nombreux paramètres biologiques et comportementaux des parasites et de leurs hôtes. Les différents types de traitement peuvent être programmés de façon à étudier leur impact dans le temps. La dynamique confirme le rôle clé joué par les puces adultes sur encore émergées et contenues dans le cocon. Seuls des traitements insecticides rémanents appliqués régulièrement sur l’hôte permettent d’obtenir un contrôle des populations parasitaires. Une combinaison avec des IGR (Insect Growth Regulator) accélère la décontamination de l’habitat et peut permettre une disparition des parasites si ces derniers ne sont pas réintroduits dans le milieu. L’association aux mesures complémentaires de type aspiration accélère le processus de décontamination, mais celles-ci n’ont aucun impact si elles sont isolées. Les traitements insecticides ponctuels ne permettent pas de réduire la population parasitaire, même s’ils sont faits fréquemment. La lutte insecticide dans l’environnement n’apparaît pas comme un moyen de lutte adéquat. Ce modèle pourrait être utilisé pour tester différents moyens de lutte intégrée, selon différentes situations : nombre d’hôtes, sorties fréquentes, incidence saisonnière.

**MOTS CLÉS:** *Ctenocephalides felis*, modélisation mathématique, dynamique de population, insecticide, IGR.

**INTRODUCTION**

The biology of *Ctenocephalides felis felis*, the cat flea, is well known today through the work of various teams, in particular those of Dryden (Bowman, 1999; Beugnet & Bourdeau, 1999; Dryden & Rust, 1994; Kaufman, 1996). Domestic carnivores can be infected by several types of flea (*Ctenocephalides felis*, *Ctenocephalides canis*, *Pulex irritans*, *Archeopsylla erinacei*, *Spilopsyllus cuniculi*, *Xenopsylla cheopis*, *Ceratophyllus gallinae* and *Leptopsylla segnis*) but, with few exceptions it is the cat flea, *Ctenocephalides felis felis*, that is involved in 90 % of cases of infestation of pet cats and dogs. The fleas of rodents, small carnivores, wild insectivores or birds are observed more rarely (Beaucournu & Ménier, 1998; Franc *et al.*, 1998; Kettle, 1995).

*Ctenocephalides canis* is observed throughout Europe, but its frequency varies from 2 to 30 % of cases, depending on the country. This species is palearctic in origin, adapted to colder zones. *C. felis* is African in origin and is thought to have been imported into Europe by returning crusaders. It then spread throughout the world in the wake of human migration. The sub-species present in Europe is *C. felis felis* (Beaucournu & Ménier, 1998). The cat flea is not particu-
larly specific and can feed off the blood of a variety of mammals (carnivores, rabbits, ruminants or humans). It is not uncommon for pet owners to be bitten by fleas (Ménier & Beaucournu, 1999). The flea is the most common ectoparasite, infesting both rural and urban environments and can be observed throughout the year, although infestation tends to be greatest from spring through to autumn (Dryden & Rust, 1994).

*Ctenocephalides felis felis* is a small, wingless, insect, 2 to 4 mm in length, reddish-brown to black in colour. The body is laterally compressed, which allows for easy movement through fur, and the hind legs are highly developed, powerful for jumping. The average distance jumped by *Ctenocephalides felis felis* is 20 cm (varying from 2 to 48 cm), and 30 cm (from 3 to 50 cm) for *Ctenocephalides canis*. As to height, this is approximately 15 cm, with a maximum of 25 cm for *C. canis* (Cadiergues et al., 2000).

Knowledge of the flea life cycle is essential to devising effective prevention of infestation. A number of commonly accepted views are false, in particular that the adult flea is a transient parasite, present on the host cat or dog only when it requires a fresh blood meal. Adult fleas tend to stay permanently on the same animal (Kettle, 1995; Kramer & Mencke, 2001). When they fall off, they can only survive approximately three to five days in the home environment. A very small proportion of fleas present on a carnivore will change host and infest another animal. The risk of contamination through direct contact between animals, such as in a vet’s waiting room, although often mentioned, is low (Franc & Cadiergues, 1997).

It is not always easy for a pet owner to see when an animal is infested with fleas. A number of experiments have shown that they are only seen on about one third of animals infested. The use of a fine comb increases the sensitivity of the search and seeing them on the animal is infested. The use of an insecticide. Host animals can be infected by moving into an infested environment, starting the life cycle going, then any dog or cats who reside there will also become infected. Environmental conditions play a part in flea ecology and in the chronology of the flea life cycle. All stages are sensitive to drying and a relative humidity rate of 85 % is optimal. Temperature accelerates or slows development, a minimum of 22°C seemingly being required, with an optimum of 25-26°C. On the other hand, a temperature above 30°C will decrease the life span of adults. In winter, an outside temperature approaching 0°C will cause the death of both larvae and pupae. If temperature in the house is 19°C, the life cycle is considerably slowed and only pre-emerged adults will remain.

These factors explain the presence of fleas all year round, and the high, given their prolificacy and the rapidity of the development cycle, there is a population explosion as soon as fine weather appears. In conditions of sufficient humidity, *Ctenocephalides felis* has a life cycle which lasts 14 days at a temperature of 29°C and 140 days at 13°C (Dryden & Rust, 1994). Adult fleas will emerge from the cocoon when submitted to various stimuli. A passing shadow, footsteps, vibrations, can all cause fleas to leave the cocoon, as can running a vacuum cleaner over the floor. So it can be of great interest to do this before, or just after, treatment of the home environment with an insecticide. Host animals can be infected by moving into an infected environment, whether outside, season permitting (walks, holidays), or inside (another pet owner’s home environment). Often cats will “import” fleas into a home environment, starting the life cycle going, then any dog or dogs who reside there will also become infected. Given biological and ecological data, a mathematical model of a flea population in a defined environment, such as an apartment where a dog or cat is present, can be envisaged. If the results tally with *in vivo* and

Larvae feed on almost any organic debris, in particular dried adult flea fecal matter. Larvae prefer humidity and avoid light. They can move horizontally about 20 cm, in order to take cover (under a chair, for example, or to the bottom of carpet and rug fibres) (Robinson, 1995). After three stages of larval development and in a period of anything from one week to one month, each 6 to 7 mm stage 3 larvae will spin a cocoon in which the pupae will develop into the adult stage in about 10 days. If conditions are favourable, i.e. if animals are present in the environment, the adults will emerge rapidly. If conditions are not favourable, however, the young adult fleas in the cocoon, called non-emerged adults, can still survive for several months (on average 150 days) protected by the silk cocoon (Beugnet & Bourdeau, 1999; Dryden & Rust, 1994; Kramer & Mencke, 2001). These non-emerged adults are a significant source of parasites immediately available should a host animal happen to pass in the vicinity. In addition, they are relatively protected from insecticides. Newly emerged fleas actively look for host animals and can survive about a week without a meal.

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in vitro biological studies, this model would then permit simulation of the impact of various types of control measures – chemical (with insecticides or hormone inhibitors), or mechanical – and an appraisal of combinations of these types of control measures.

MATERIAL AND METHODS

Given the known biology of the different stages of the flea life cycle, the mode of development, and a chronological sequence established in terms of the surroundings and conditions in the home environment, it is possible to reproduce this cycle in diagram form.

GENERAL DESCRIPTION OF MODEL AND DEFINITION OF VARIABLES

We first of all considered the five variables representing the stages of flea development:
- \( A(t) \): total number of adult fleas at time period \( t \);
- \( WL(t) \): total number of fleas at egg and larval stage at time period \( t \);
- \( N(t) \): total number of fleas at nymph stage, as a function of time;
- \( NL(t) \): total number of fleas at latent nymph stage (i.e. that will not immediately evolve into newly emerged adults), as a function of time;
- \( UFA(t) \): total number of pre-adult fleas, as a function of time.

Chronological parameters were then assigned for the various stages in the flea life cycle:
- \( T_1 \): period of embryo and larval stages of development \([T_1 = \text{from 10 to 30 days, depending from the climate}]\);
- \( T_2 \): period of nymph stage of development \([T_2 = \text{four days}]\);
- \( T_3 \): period during which latent nymphs do not die \([NL_1(t)] \); \([T_3 = 90 \text{ days}]\);
- \( T_4 \): period during which latent nymphs die \([NL_2(t)] \); \([T_4 = 150 \text{ days}]\);
- \( T_5 \): average time of persistence at UFA stage \([UFA = \text{pre-emerged fleas, still in cocoon}] \); \([T_5 = \text{one day}]\).

Biological parameters concerning rates of mortality, prolificacy, sex ratio, were also assigned at various stages:
- \( \mu_1 \): mortality rate for adults;
- \( \mu_2 \): mortality rate for eggs/larvae/nymphs;
- \( \mu_3 \): mortality rate for latent nymphs;
- \( \mu_4 \): mortality rate for UFA;
- \( x_1 \): number of adult fleas brought in from outdoors (introduced into the model);
- \( x_2 \): number of UFA's brought in from outdoors;
- \( r \): density-dependent excess mortality rate;
- \( k \): maximum receptation capacity, dependent on host animal;
- \( \text{fem} \): proportion of female in adult population;
- \( \lambda \): prolificacy;
- \( \alpha \): percentage of fertile eggs;
- \( \beta \): percentage of success rate for development of immature stages;
- \( \gamma \): percentage of latent nymphs surviving after time \( T_3 \), during \( T_4 \);
- \( v \): rate of passage of latent nymphs to UFA, dependent on presence of host animal;
- \( \delta \): percentage of success of UFA's.

These parameters are summarized in Table I. They enable a progressive model to built.

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**Diagram:**

![Kinetic diagram of model.](image-url)

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Mémoire
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<td>WL</td>
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<td>µ4</td>
<td>1/day</td>
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<td>x2</td>
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<td>r</td>
<td>-</td>
<td>0.7</td>
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<td>K</td>
<td>flea</td>
<td>300</td>
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<td>fem</td>
<td>-</td>
<td>2/3</td>
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<tr>
<td>Prolificity</td>
<td>λ</td>
<td>N eggs/female</td>
<td>27*(IGRto*IGRvo)</td>
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<td>Success rate egg/larvae development</td>
<td>α</td>
<td>-</td>
<td>(alf [...Psol]<em>Am</em>IGRe)</td>
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<td>β</td>
<td>-</td>
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<td>-</td>
<td>0.25*INSr</td>
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<td>v2</td>
<td>-</td>
<td>2/3</td>
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<td>-</td>
<td>1<em>Shamp</em>INSr*INSnr</td>
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<td>Hostcomp</td>
<td>-</td>
<td>0</td>
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<td>Season</td>
<td>-</td>
<td>0</td>
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<td>Modulating function of floor cover rug/carpet on population</td>
<td>Floor</td>
<td>-</td>
<td>0 % 0.2 0.7 1</td>
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<tr>
<td>Modulating function of home environment on success of hatching</td>
<td>alf [°C, % H2O]</td>
<td></td>
<td>&lt; 30% [30-60%] [60-75%] [75-85%] ≥ 85</td>
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<tr>
<td>Treatment parameters</td>
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<td>Am</td>
<td>-</td>
<td>0</td>
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<td>Efficacy of insecticide on environment</td>
<td>INSr</td>
<td>-</td>
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<td>Efficacy of IGR on environment</td>
<td>IGRto</td>
<td>-</td>
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<td>Efficacy of non persistent insecticide</td>
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<td>Efficacy of IGR on host animal, topical formulation</td>
<td>IGRvo</td>
<td>-</td>
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<tr>
<td><strong>Table I.</strong> Principal parameters of the model: symbol, unit and value.</td>
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</table>

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DYNAMICS OF Ctenocephalides felis populations

Formulation of model is presented on Figure 1, in which:
- WL: represents the population of eggs and larvae. They have a quick evolution, $T_1$ (from 10 to 30 days), before becoming nymphs;
- N: nymphs, with a mortality rate of $\mu_2$ which could not be distinguish from the global mortality of eggs and larvae. The nymph will evolve within a $T_2$ time (four days) to become a NL stage which includes a future adult flea. The nymph population is subdivided in NL1 and NL2, which are the final nymph stages, including a non-emerged flea;
- NL1: late nymph or cocoon which includes a pre-adult stage = non emerged flea. NL1 could evolve very quickly by releasing newly emerged fleas, especially in case of the presence of host. Nevertheless, in case of absence of the host, NL1 will become latent, which corresponds to the NL2 status;
- NL2: latent pre-emerged fleas inside the cocoon, which represents a stock of future flea in the environment. These stages are waiting for host (dog/cat) to emerge and become newly emerged fleas (= UFA);
- NL1 + NL2 are living during a minimum of $T_3$ time, which is around 90 days. After this delay, the rate of mortality increase and the NL population is decreasing during a $T_4$ time (in average 150 days);
- UFA: Newly emerged fleas, which could come from NL1 (quick appearance) or from the NL2 stock (quiescent pre-emerged fleas present in the environment and waiting for a stimulus). The newly emerged fleas have to find and jump very fast on their host as they cannot stay alive a long time without a first blood meal. The average time of life before becoming Adult parasite stage or dying is $T_5$ (from one to three days, one in the model below).

THE MODEL IN EQUATION FORM

Given the diagram and the parameters set, the model can be formulated into an equation. The equations describing the flea life cycle as a function of time are written as follows:

\[
\begin{align*}
\frac{dA(t)}{dt} &= \delta A(t) + k(t) - rA(t) - \mu_A(t) \\
\frac{dWL(t)}{dt} &= F(t) - e^{-\mu_L}F(t-T_1) - \mu_2 WL(t) \\
\frac{dN(t)}{dt} &= e^{-\mu_L}F(t-T_1) - e^{-\mu_N}F(t-T_1-T_2) - \mu_2 N(t) = e^{-\mu_L}F(t-T_1) - \beta F(t-T_1-T_2) - \mu_2 N(t) \\
\frac{dNL_1(t)}{dt} &= \beta F(t-T_1-T_2) - \beta F(t-T_1-T_2-T_3)e^{-\mu_N} - \nu NL_1(t) \\
\frac{dNL_2(t)}{dt} &= \beta F(t-T_1-T_2-T_3)e^{-\mu_N} - (\nu + \mu_4) NL_2(t) - \beta F(t-T_1-T_2-T_3-T_4)e^{-\mu_N}e^{-\mu_L}e^{-\mu_L} \\
\frac{dUFA(t)}{dt} &= \nu(NL_1(t) + NL_2(t)) - \left(\frac{\delta}{T_3} + \mu_4\right) UFA(t)
\end{align*}
\]

where:

\[
F(t) \alpha \lambda A(t) (2)
\]

where terms of mortality $\mu$ are equal to:

\[
\begin{align*}
\mu_2 &= -\ln \beta \\
\mu_3 &= -\ln \gamma \\
\mu_4 &= 1 - \delta 
\end{align*}
\]

and where $T_1$ is the development time for eggs and larvae so that:

\[
T_1 = (1 - \beta) \times T_{1\text{max}}
\]

$T_2$ is the development time for nymphs.

As to nymphs-in-waiting, at the end of a maximum time $T_3 + T_4$, only $\gamma$ remain.

With $k$ dependent on the host animal, and $\nu$ and $\delta$ dependent on the presence of the host, $\nu$ and $\delta$ are nil in the absence of a host.

In the case where the host is absent, adult fleas disappear, whereas the immature stages continue their development to the latent nymph stage.

The numerical values are as follow.

The following initial conditions have been selected for this part of the model:

\[
\begin{align*}
A(t=0) &= 20 \\
WL(t=0) &= 0 \\
N(t=0) &= 0 \\
NL_1(t=0) &= 0 \\
NL_2(t=0) &= 0 \\
UFA(t=0) &= 0
\end{align*}
\]

We thus start with a healthy environment and the introduction of an animal infested with 20 adult fleas. We then consider the existence of in-flows which will interfere with the base model. These flows consist of the entry of adult fleas subsequent to the outdoor movements of the host animal (classic for the cat, for example), but also the possibility of nymphs/pupae being introduced by an owner.
DEFINITION OF IN-FLOWS

Two in-flows, independent of the development cycle, can be distinguished:

An adult flow \( k_I \) originating from outdoors that the host animal brings in to the home environment after having been outdoors. It should be noted that \( k_I \) is a function of time. \( k_I(t) = x_1 \) in the case where there is a host going outdoors in the warm season (spring to autumn), otherwise \( k_I(t) = 0 \).

This is summarized as:

\[
k_I(t) = x_1 \times \text{Season} \times \text{HostComp} \times \text{Host}
\]

where \( \text{Season} \) and \( \text{HostComp} \) are such that:

\[
\text{Season} = \begin{cases} 
0 & \text{if } t \in [0, 180] \\
1 & \text{if } t \notin [0, 180]
\end{cases}
\]

\[
\text{HostComp} = \begin{cases} 
1 & \text{if host goes out} \\
0 & \text{if host does not go out}
\end{cases}
\]

DEFINITION OF STATE OF THE HOME ENVIRONMENT

To these equations, we then added the effect of the home environment on the successful development of embryos and larvae. The success rate for this period is indicated using the parameter \( \beta \).

In the home environment, two elements are necessary for the survival of individual fleas: (i) that floor coverings are suitable for larva development, i.e. rugs, carpets and old parquet flooring, and (ii) conditions of temperature and humidity in the home environment. These two parameters are summarized as follows:

- \( \text{sol} \): modulating function of floor coverings rugs/carpet on the population;
- \( \beta \): modulating function of home environment on the success of development;

with, for the function \( \text{sol} \), the following values according to the type of covering:

\[
\text{sol} = \begin{cases} 
0 & \text{if } 0 \% \\
0.4 & \text{if } 10 \% \text{ to } 20 \% \\
0.7 & \text{if } 20 \% \text{ to } 50 \% \\
1 & \text{if } 50 \% \text{ to } 100 \%
\end{cases}
\]

and for function \( \beta \), resulting from the combined effect of temperature \( T \)°C and humidity \( H_2O \), the values are:

\[
\begin{array}{c|cccc}
T \degree C & < 30 & \text{[30-60]} & \text{[60-75]} & \text{[75-85]} & \geq 85 \\
\hline
\% H_2O & 0 & 0 & 0.3 & 0.4 & 0.5 \\
\hline
[0-13] & 0 & 0.5 & 0.6 & 0.7 & 0.8 \\
[13-22] & 0 & 0.7 & 0.8 & 0.9 & 1.0 \\
[22-30] & 0 & 0.9 & 1.0 & 1.1 & 1.2 \\
[30-40] & 0 & 1.0 & 1.1 & 1.2 & 1.3 \\
\geq 40 & 0 & 1.1 & 1.2 & 1.3 & 1.4 \\
\end{array}
\]

Parameter \( \beta \) thus becomes a function which depends on conditions in the home environment and can then be written:

\[
\beta = \alpha \times \text{sol} \times \beta [T \degree C, \% H_2O]
\]
Dynamics of *Ctenocephalides felis* populations

Fig. 2. - $A_s(t)$ as a function of time.

The passage of the vacuum cleaner in the home environment has an effect on the dynamics of the flea population, but only on the eggs/larvae and UFA segment since the latter cannot survive being sucked up and die in the vacuum cleaner bag. They are therefore removed from the equation. This action affects both eggs, larvae and UFA. $\mu_2$ is therefore increased, but only in the WL stage, although this does have an effect on the other segments.

$$\mu_2 = \frac{\ln \beta}{T_1 + T_2} - \ln A_s(t)$$

(9)

In addition, the action of the vacuum cleaner on the UFA’s is:

$$\frac{dUFA(t)}{dt} = k_2(t) + n (NL_1(t) + NL_2(t))$$

$$- \left( \frac{\delta}{T_3} + \mu_a \right) UFA(t) - \frac{(1 - A_s)}{\Delta t} UFA(t)$$

(10)

with $\Delta t$ being the time step. Without the vacuum cleaner there is no further mortality. On the other hand, when the vacuum cleaner is 100 % effective, there will be a maximum mortality of $-\frac{1}{\Delta t}$ corresponding to total and instant disappearance of the UFA population.

• Application of insecticide or IGR to the home environment premises

It is possible to treat the home environment premises so as to eliminate the fleas. Two types of treatment are usually used: an insecticide, or an IGR (Insect Growth Regulator) (Carlotti & Jacobs, 2000; Kramer & Mencke, 2001).

Insecticide

As with vacuuming, insecticides applied to the environment will only have a temporary action. They affect $INS(t)$ which represents a modification in the success rate of the UFA stage to the adult stage connected with the action of the insecticide:

For $t = \eta \tau$, $INS(t) = 1 - e_2$

For $\neq \eta \tau$, $INS(t) = 1$

(11)

where efficacy $e_2$ is then equal to 80 %.

This treatment increases mortality and therefore decreases the percentage of success at the UFA stage, $\delta$. From that point, when $\delta$ is not nil, it is no longer constant and becomes:

$$\delta(t) = 0.4 \times INS(t)$$

(12)

Insect Growth Regulator

Treatment of the home environment premises with an IGR is not a temporary action, but one which extends in time. It can thus be summarized as function of an insecticide and larvae survival rate and is expressed as follows:

For $\eta \tau \leq t < \eta \tau + t_1$, $IGR(t) = 1 - e_3$

Or otherwise, $IGR(t) = 1$

(13)

where $e_3 = 60 \%$ for $t_1 = 60$ days.

The IGR acts directly on the potential survival of eggs and larvae. There is therefore additional mortality at the WL stage of:

$$- \ln IGR(t)$$

(14)

• Application of insecticide to host animal

Two types of insecticide can be used to treat the animal directly for fleas: persistent insecticides, $INSr(t)$ (the most recent being the “spot on” type), and non-persistent, $INSnr(t)$, the older types (such as powders, lotions and shampoos).

Non-persistent insecticides

Non-persistent insecticides are applied directly to the host animal and kill all adult fleas on the host. Duration of efficacy is short and action rapid (Franc & Cadiergues, 1995). $INSnr$, which represents modification in the attractiveness and reception of the host connected with the non-persistent insecticide, is such that:

For $\eta \tau \leq t < \eta \tau + 2$, $INSnr(t) = 0$

Or otherwise, $INSnr(t) = 1$

(15)

Action $INSnr$ causes additional mortality of adult fleas:

$$\frac{1 - INSnr(t)}{\Delta_1}$$

(16)

and a decrease in the success of UFA’s:

$$\delta(t) = 0.4 \times INS(t) \times INSnr(t)$$

(17)

Likewise, fleas from the outdoors are then:

$$k_1(t) \times INSnr(t)$$

(18)

Persistent insecticides

Persistent insecticides are applied directly to the host animal. They destroy part or all adult fleas on the host for a more or less long period of time (Cadiergues et al., 2001; Dryden et al., 2000; Schencker et al., 2003).

In this study, we considered that the efficacy $e_4$ of this type of insecticide was 100 % for a duration of 45 days for dogs, and 30 days for cats. After this, the effect disappears over a period of 60 days, whichever type of animal is the host animal. Consequently, we will
consider $I_{SR}(t)$, representing the change in adult mortality connected with the persistent insecticide, as a function varying in time (Fig. 3).

Action $I_{SR}(t)$ is such that for a maximum, 100 % effect, $I_{SR}(t) = 0$ and the mortality rate is maximum $\frac{1}{\Delta t}$. This is equivalent to saying that in one time step the adult population disappears.

When there is no effect, $I_{SR}(t) = 1$ and the mortality rate is $d_f$. Whence, the mortality rate is worth:

$$\left(1 - \frac{1}{\Delta t}\right)I_{SR}(t)I_{SR}(t) + \frac{1}{\Delta t}$$

- Application of an IGR to a host animal

In addition to IGRs which treat the home environment premises, two different types of external pest control products are available on the market: an oral IGR, $IGR_{vo}$, and a topical IGR, $IGR_{to}$ (Jacobs et al., 1997; Jacobs et al., 2001; Schipstone & Mason, 1995). The latter can be combined with a persistent insecticide, $I_{SR}$.

Administration of an oral IGR

An oral IGR will affect egg production and development for 30 days followed by a rapid decrease; it therefore has a step function. This type of treatment can then be described using the following expression:

for $\eta \tau \leq t < \eta \tau + 30$, $IGR_{vo}(t) = 0$

or otherwise, $IGR_{vo}(t) = 1$ (19)

Where $IGR_{vo}(t)$ represents the modification to egg production connected with the oral IGR.

We can consider that treatment with IGRs reduces the prolificacy of female fleas. Thus, the oral IGR can be entered as $\lambda$. We therefore obtain:

$$\lambda = 27 \times IGR_{vo}$$ (20)

Administration of a topical, spot-on IGR

This type of action, used in conjunction with other types of treatment, blocks nymph transformation. The period during which efficacy is 100 % is 60 days for both dogs and cats. The period of decrease is also identical in both species (60 days). $IGR_{to}$ represents

the modification of the success rate of larvae development in connection with the topical IGR (Fig. 4). Therefore, the number of larvae reaching the latent nymph state at time $t$ is $\beta IGR_{to}(t)F(t - T_1)$.

Application of a combination of insecticide + IGR (methoprene) on host animal.

Here, there is the combined action of $I_{SR}(t) \neq 1$ and $IGR_{to}(t) \neq 1$.

RESULTS

Simulations of flea population dynamics in a closed home environment (apartment with presence of a cat) and of the incidence of classic treatment strategies on these dynamics. For all simulations:

A (t) in red

WLN (t) in blue

NL (t) = NL_{1}(t) + NL_{2}(t) in green

Treatments in black.

The software programme used to carry out the simulations was: Berkeley Madonna, Version 8.0.1, ©1997-2000 Robert I. Macey & George F. Oster.

OBSERVATION OF MODEL DYNAMICS

- Flea population dynamics with no in-flow of fleas. Base line: one cat infested with 20 adult fleas (Fig. 5) Conditions in home environment:

  - favourable humidity and T °C and important floor covering (alf[3,4]*sol2);
- host animal permanently present, no movement outdoors; 
in an environment: (bet[3,3]*sol4).

Host animal permanently present and no movement outdoors (infestation at T0 of 20 adult fleas, no infestation of the home environment premises to begin with).

- Mechanical action by vacuum cleaner (Figs 7 & 8) One session of vacuuming at D120 (Fig. 7). We consider that the vacuum cleaner acts on eggs, larvae and UFA’s but not on latent nymphs with mean efficacy of 40 % for all stages.

Vacuuming apartment once a week (Fig. 8)

- Simultaneous application of insecticide and IGR to home environment premises (Figs 9 & 10) 
Single application at D 120 (Fig. 9)
Single application to home environment premises twice a year (Fig. 10)

- Application of non-persistent insecticide to host animal. Single treatment applied once a week for four weeks (Fig. 11)

- Application of persistent insecticide to host animal (Figs 12 & 13) 
Single treatment applied at D 120 (Fig. 12).
Repetition of treatments at D 120, D 150 and D 180, or treatment over three successive months (Fig. 13).

- Application of IGR on host animal (Figs 14-17) 
Oral administration of IGR al at D 120, D 150 and D 180, or treatment over three successive months (Fig. 14).
Fig. 11. - Application of non-persistent insecticide to host animal: single treatment applied once a week for four weeks (Fig. 11).

Fig. 12. - Application of persistent insecticide to host animal: single treatment applied at D 120.

Fig. 13. - Application of persistent insecticide to host animal: repetition of treatments at D 120, D 150 and D 180, or treatment over three successive months.

Topical administration of IGR: administration at D 120 and at D 180 (Fig. 15).

Trial combining non-persistent insecticide + oral IGR: use of non-persistent insecticide three days running each week for one month, with parallel oral administration of IGR (Fig. 16).

Trial combining non-persistent insecticide + oral IGR: use of non-persistent insecticide three days running each week for one month, with parallel oral administration of IGR for three months (Fig. 17).

• Treatment of host animal combining persistent insecticide and topical (spot-on) IGR (Figs 18-20)

Treatment of host animal at D 120, D 180 and D 240, which corresponds to application of this association every two months (Fig. 18).

Fig. 14. - Oral administration of IGR on host animal at D 120, D 150 and D 180, or treatment over three successive months (Fig. 14).

Fig. 15. - Topical administration of IGR on host animal: administration at D 120 and at D 180.

Fig. 16. - Trial combining non-persistent insecticide + oral IGR on host animal: use of non-persistent insecticide three days running each week for one month, with parallel oral administration of IGR.

Fig. 17. - Trial combining non-persistent insecticide + oral IGR on host animal: use of non-persistent insecticide three days running each week for one month, with parallel oral administration of IGR for three months.
Fig. 18. – Treatment of host animal combining persistent insecticide and topical (spot-on) IGR at D 120, D 180 and D 240, which corresponds to application of this association every two months.

Fig. 21. – Dynamics of model with regular outdoor movements of host animal and in-flow of fleas: use of the association persistent insecticide + spot-on IGR at D 120, D 180 and D 240.

Fig. 19. – Treatment of host animal combining persistent insecticide and topical (spot-on) IGR at D 120, D 180, D 240 and D 300, i.e. four treatments with two months between treatments.

Fig. 22. – Dynamics of model with regular outdoor movements of host animal and in-flow of fleas: use of the association persistent insecticide + spot-on IGR at D 120, D 180, D 240 and D 300.

Fig. 20. – Treatment of host animal combining persistent insecticide and topical (spot-on) IGR every two months i.e. three treatments (at D 120, D 180 and D 240) but with regular vacuum cleaning of the apartment once a week.

Fig. 23. – Dynamics of model with regular outdoor movements of host animal and in-flow of fleas: use of the association persistent insecticide + spot-on IGR at D 120, D 180, D 240 + regular vacuum cleaning of the home environment premises once a week.

Treatment of host animal at D 120, D 180, D 240 and D 300, i.e. four treatments with two months between treatments (Fig. 19).

Treatment of host animal every two months i.e. three treatments (at D 120, D 180 and D 240) but with regular vacuum cleaning of the apartment once a week (Fig. 20).

- Dynamics of model with regular outdoor movements of host animal and in-flow of fleas (Figs 21-23)

Conditions in home environment:
- right humidity and T °C and important floor covering (bet[3,3]sol4);
- cat permanently present in home environment but with movement outdoors (K = 200).

Use of the association persistent insecticide + spot-on IGR at D 120, D 180 and D 240 (Fig. 21).

Use of the association persistent insecticide + spot-on IGR at D 120, D 180, D 240 and D 300 (Fig. 22).

Use of the association persistent insecticide + spot-on IGR at D 120, D 180, D 240 + regular vacuum cleaning of the home environment premises once a week (Fig. 23).
DISCUSSION

Mathematical models are widely used to study certain populations of insects such as the tsetse fly, vector of trypanosomes, or mosquitoes, vectors of malaria. In veterinary medicine, a number of authors have been interested in modelling tick populations (Beugnet et al., 1998) but no model, simple and able to be used by everyone, has been devised for modelling flea populations. The model described in the present paper takes into account the biological and chronological parameters that have been published concerning Ctenocephalides felis (Dryden & Rust, 1994; Kramer & Schencke, 2001). The model confirms the exponential increase in infestation of the home environment premises and of the host animal. This very flexibly constructed model can handle variations of numerous parameters: seasonal variations, impact of type of floor covering, behaviour of host animal who may leave the home premises for variable periods of time, leaving the contaminated premises in an “on-hold” state. The incidence of the various types of treatment can also be easily factored in.

The model confirms the major role of nymphs or latent cocoons containing the newly emerged future fleas (Beugnet & Bourdeau, 1999). It is the resistance of this stage of the cycle that makes its mark on population dynamics and makes treatment difficult. For instance, vacuum cleaning of the home premises alone does not entirely solve the problem of contamination of the home environment even though it is highly effective. No chemical treatment carried out as a one-off will eliminate the flea population since the in-flow of new fleas is not considered. The use of an Insect Growth Regulator along with an insecticide in the market. The impact of an eventual anti-flea vaccine could also be tested using this model.

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